Exercise 58

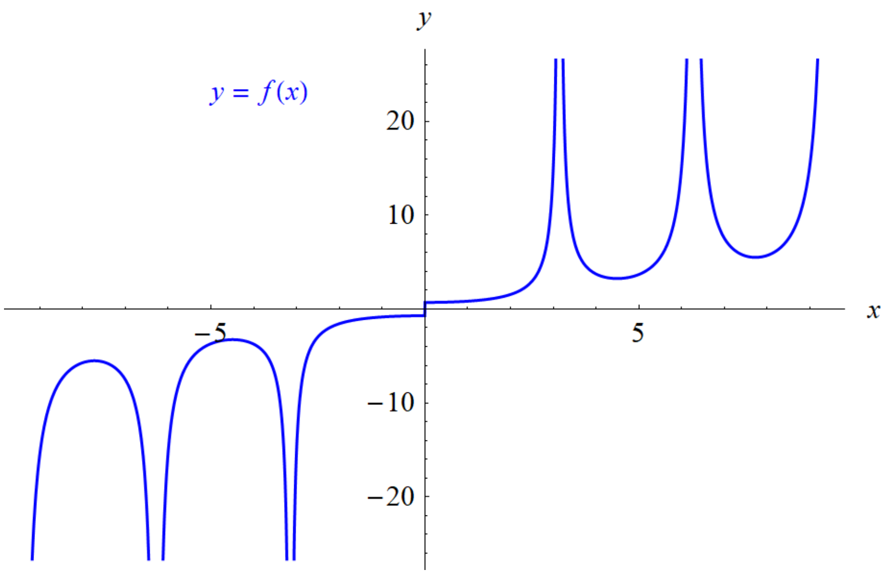
Let $f(x) = \dfrac{x}{\sqrt{1 - \cos 2x}}$.

(a) Graph f. What type of discontinuity does it appear to have at 0?

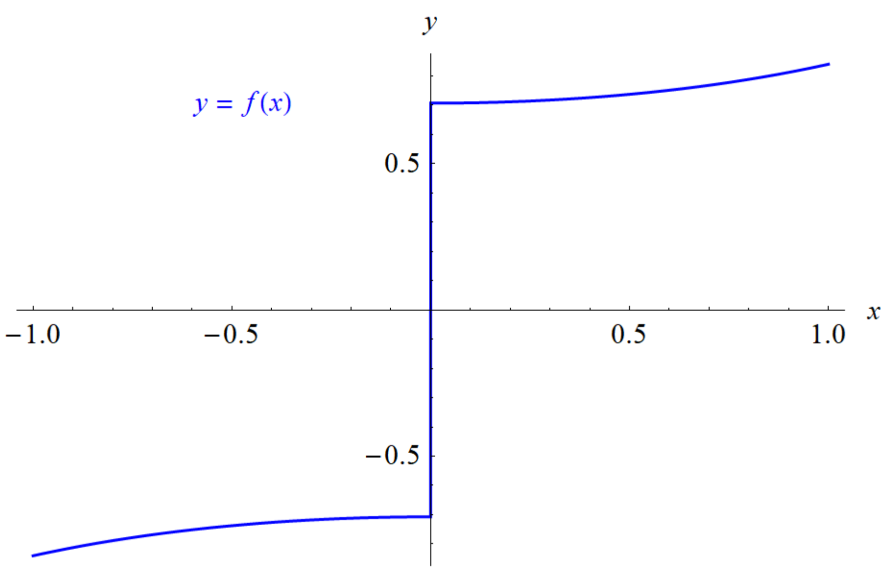
(b) Calculate the left and right limits of f at 0. Do these values confirm your answer to part (a)?

Solution

Below is a graph of $f(x)$ versus x.



It seems to have a jump discontinuity at $x = 0$. Another graph is shown for $-1 < x < 1$ to illustrate this better.



The aim is to calculate the left- and right-hand limits as $x \to 0$. Rewrite $f(x)$ first.

$$f(x) = \frac{x}{\sqrt{1-\cos 2x}} = \frac{x}{\sqrt{2\sin^2 x}} = \frac{1}{\sqrt{2}}\frac{x}{|\sin x|}$$

Now evaluate the limits.

$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} \frac{1}{\sqrt{2}} \cdot \frac{x}{-(\sin x)} = -\frac{1}{\sqrt{2}} \lim_{x\to 0^-} \frac{1}{\frac{\sin x}{x}} = -\frac{1}{\sqrt{2}} \approx -0.707$$

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} \frac{1}{\sqrt{2}} \cdot \frac{x}{(\sin x)} = \frac{1}{\sqrt{2}} \lim_{x\to 0^+} \frac{1}{\frac{\sin x}{x}} = \frac{1}{\sqrt{2}} \approx 0.707$$